

**DESIGNING AND IMPLEMENTING GREY RELATIONAL ANALYSIS FUNCTION
USING MATLAB****Yrd.Doç.Dr. Mehmet ÖZÇALICI**Kilis 7 Aralık University, Faculty of Economics and Administrative Sciences, International Trade
and Logistics Department, Kilis/Turkey**ABSTRACT**

The aim of this study is to design a function that will easily calculate Grey Relational Grades by calling a function in MATLAB environment. Grey Relational Analysis (GRA) is a member of Grey System and it has successfully applied as a decision support tool in many areas. It is a nonparametric tool and it is possible to make decisions by using relatively small dataset which make it an attractive method. In this study, first the theory of the Grey Relational Analysis is presented and secondly the steps of the GRA function is explained in detail. In addition, a problem, which is solved and published in literature, is solved by using the function presented in the study. Designing a function for Grey Relational Analysis will enable researchers to use it in different areas and for different purposes.

Keywords: Grey Relational Analysis, MATLAB**1. INTRODUCTION**

It is possible to define the decision making as a procedure to find the best alternative among a set of feasible alternatives (Park et al 2011). Multi-Criteria Decision Making (MCDM) tools are formal approaches to support the decision making process by presenting the ranking among alternatives (Athawale and Chakraborty, 2008). In MCDM tool family, there are various kind of tools available such as Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Fuzzy TOPSIS, ViseKriterijuska Optimizacija I Komoromisno Resenje (VIKOR), Analytical Hierarch Process (AHP), Fuzzy AHP and Grey Relational Analysis.

In literature different kind of business problems are solved by MCDM tools. A few of them are supplier selection, automobile selection, cell phone and personal computer selection, business administration school selection etc. In addition, MCDM tools are employed for different purposes such as for forecasting the business failure, for benchmarking tool, for evaluating the performance of business enterprises.

A few software is designed to implement MCDM tools. However, it is not possible to use them in hybrid models. In other words, if the functions are moved to a higher platform where it is possible to implement complex analysis, the actual performance of GRA will be analyzed in a deeper manner. The purpose of this study is to design a series of functions that will implement the Grey Relational Analysis calculation steps in MATLAB environment. The MATLAB can be defined as both a programming language and a scientific software. Users can just call a single function to implement complex calculations. However, there is no readily available GRA function. The function presented in this study can increase the popularity of GRA in Business and Administration Sciences.

2. LITERATURE REVIEW

(Athawale and Chakraborty, 2008) applied GRA to solve supplier selection problems. They utilized two real world problems that is solved by other researchers in literature with different methods. At the end of the study, they reported that, GRA approach produced the same ranking with previous methods. Vahdani et al (2008) presented a supplier selection problem and solved the problem with

Balancing and Ranking Method. At the end of the study, it is reported that their methodology is efficient in solving the supplier selection problem.

Peker and Baki (2011) measured the performances of three insurance company operating in Turkey. They calculated 10 financial ratios of three insurance companies. At the end of the study, it is reported that GRA enabled to focus on ten financial ratios instead of single ratio.

Baş ve Çakmak (2012) applied GRA as forecasting the failure of firms. They calculated 21 financial ratios as criteria. They also employed logistic regression as forecasting tool. It is reported that; it is possible to obtain reliable results by using GRA. Çakmak et al (2012) applied grey relational analysis and correspondence analysis in a business enterprise. At the end of the study, it is reported that their proposed method is an effective method as decision-making.

Ayrıçay, Özçalıcı ve Kaya (2013) applied Grey Relational Analysis as a financial benchmarking tool and presented an application for BIST30 non-Financial Firms. They brought together 23 financial ratios for 21 non-financial firms and applied GRA. At the end of the study, it is reported that GRA can be used effectively for benchmarking proposes. Şişman ve Eleren (2013) applied grey relational analysis and ELECTRE methods for selecting an optimal automobile. 10 criteria for 11 automobile models are presented in the study. At the end of the study, it is reported that two models produced different ranks for the same problem. Bektaş ve Tuna (2013) measured the performance of firms operating in Borsa Istanbul by using GRA. They calculated 6 different financial ratios by using financial statement and financial performance tables of companies. At the end of the study, they determined the company that is operating effectively.

Tayyar et al (2014) utilized Analytical Hierarchy Process and GRA to evaluate the performance of firms operating in Borsa Istanbul. They calculated 12 financial ratios belong to 11 alternatives. They applied AHP and GRA to same dataset. At the end of the study, they reported the firms with high performance and low performance.

3. METHODOLOGY

3.1. The Theory of Grey Relational Analysis (GRA)

It is possible to schematize the logic of GRA with Figure 1.

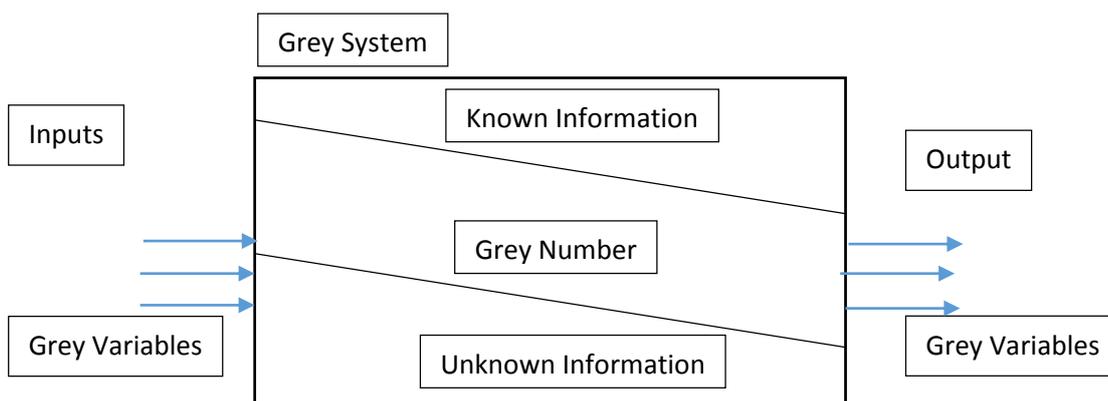


Figure 1. The Concept of a Grey System (Liu et al. 2007).

Following are the steps of the Grey Relational Analysis:

Step 1. Construction of decision matrix: The following is a decision matrix and it consists from m criteria and n alternatives.

$$X_i = \begin{bmatrix} x_1(1) & x_1(2) & \dots & x_1(m) \\ x_2(1) & x_2(2) & \dots & x_2(m) \\ \dots & \dots & \ddots & \dots \\ x_n(1) & x_n(2) & \dots & x_n(m) \end{bmatrix} \quad (1)$$

Step2. Normalization: The criteria is usually measured in different scales. That means the criteria that has bigger numerical values will dominate the analysis. To prevent this, the dataset must be normalized before using in analysis. Normalization transforms the dataset in the scale of [0,1].

Grey relational analysis has three different kind of normalization methods.

The-larger-the-better. If it is preferred larger values in a criterion, then $x_i(j)$ is transformed to a normalized value $x_i^*(j)$ by using the following equation:

$$x_i^*(j) = \frac{x_i(j) - \min x_i(j)}{\max x_i(j) - \min x_i(j)} \quad (2)$$

where, $\max x_i(j)$ represents the maximum value in criteria j and $\min x_i(j)$ represents the minimum value in criteria j . $x_i(j)$ represents the value before normalization and, x_i^* represents the value after normalization. The new value of the alternative which has the biggest (lowest) value in terms of criteria j will have a new value of 1 (0).

The-smaller-the-better. If smaller values are preferred then, $x_i(j)$ is transformed to a normalized value $x_i^*(j)$ by using the following equation:

$$x_i^* = \frac{\max x_i(j) - x_i(j)}{\max x_i(j) - \min x_i(j)} \quad (3)$$

where, $\max x_i(j)$ represents the maximum value in criteria j , $\min x_i(j)$ represents the minimum value in criteria j . $x_i(j)$ represents the value before normalization and x_i^* represents the value after normalization.

The following is a matrix that includes the normalized values of decision matrix (X_i^*).

$$X_i^* = \begin{bmatrix} x_1^*(1) & x_1^*(2) & \dots & x_1^*(m) \\ x_2^*(1) & x_2^*(2) & \dots & x_2^*(m) \\ \dots & \dots & \ddots & \dots \\ x_n^*(1) & x_n^*(2) & \dots & x_n^*(m) \end{bmatrix} \quad (4)$$

The following is the reference series that has the best values for criteria.

$$x_0^* = (x_0^*(1), x_0^*(2), \dots, x_0^*(m)) \quad (5)$$

Step 3. Compute the distance of $\Delta_{0i}(j)$, the absolute value of difference between x_0^* and x_i^* in the criteria j .

$$\Delta_{0i}(j) = |x_0^*(j) - x_i^*(j)| = \begin{bmatrix} \Delta_{01}(1) & \Delta_{01}(2) & \dots & \Delta_{01}(m) \\ \Delta_{02}(1) & \Delta_{02}(2) & \dots & \Delta_{02}(m) \\ \dots & \dots & \ddots & \dots \\ \Delta_{0n}(1) & \Delta_{0n}(2) & \dots & \Delta_{0n}(m) \end{bmatrix} \quad (6)$$

Step 4. Apply the following equation to compute grey relational coefficient:

$$\gamma_{oi}(j) = \frac{\Delta min + \delta \Delta max}{\Delta_{oi}(j) + \delta \Delta max} \tag{7}$$

where, $\Delta min = \min_j \min_i \Delta_{oi}(j)$ and $\Delta max = \max_i \max_j \Delta_{oi}(j)$ and δ represents grey coefficient and it is between $[0, 1]$.

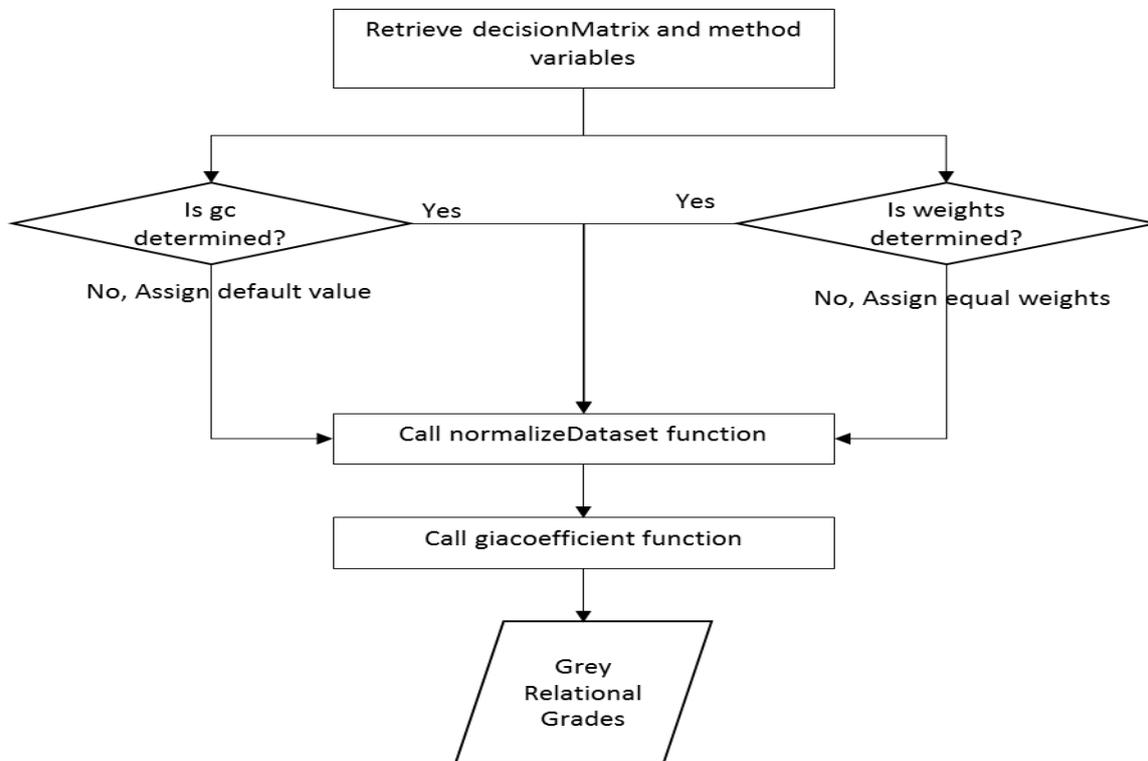
Step 5. Calculating the degree of grey coefficient with following equation:

$$\Gamma_{oi} = \sum_{j=1}^m [W_i(j) \times \gamma_{oi}(j)] \tag{8}$$

where, Γ represents the degree of grey relation, W represents the weights of criteria and γ represents grey relational coefficient.

3.2. Proposed Approach

Figure 2. Design of GRA Function



The function designed in this study needs the following inputs to calculate the GRA coefficients.

- ✓ Decision matrix: A matrix where columns represents criteria and rows represent alternatives.
- ✓ Method: This vector has number of columns equal to number of criteria. In this vector, 1 indicates higher is better characteristic and 0 represents smaller is better characteristic.
- ✓ Gc (Grey coefficient δ - optional): If this coefficient is not provided by user then it will be fixed at 0.5.
- ✓ Weight vector (optional): User can provide a vector that has the weights belong to criteria. Alternatively, it will be assumed that each criterion has same equal weight.

A sample calling of the function will be like

(generalRank = greyRelational(decisionMatrix, method,[],[])).

3.3. Designing Grey Relational Analysis Functions in MATLAB Environment

A modular approach is preferred in calculations. The MATLAB code of the main function is depicted in Figure 3. This is the main function and user only should call this function. The main function accepts input arguments, checks if *gc* and weights are available and calls other function. Finally, it calculates the general ranks and returns it as output value.

Figure3. Main Function

```
function [generalRank] = greyRelational(decisionMatrix,method,gc,weights)

% Using Grey Relational Analysis

%% Inputs
% Decision matrix is a matrix where rows represent alternatives while
% columns represents criteria.

% method is a vector which consists from 0s,1s and 2s. 1s represents higher
% is better characteristic; 0s represent smaller is better characteristic

% gc is grey coefficient if it is empty, 0.5 will be assign - optional

% weight matrix is a matrix which includes importance weights of attributes
% (criteria). If it is empty equal weights will be assigned. - optional

%% Output
% Output include grey ranks

%% Created by Mehmet ÖZÇALICI , June 2015

if isempty(gc) % if no value is assigned to gc, assign 0.5 to gc
    gc = 0.5;
end

% initialize parameters
[numCase, numVar] = size(decisionMatrix);
if isempty(weights)
    weights = ones(1,numVar)/numVar;
end

% calculate normalized dataset and reference series
[normalized, referenceSeries] = normalizeDataset(decisionMatrix,method);

% calculate gia coefficients
giaCoeff = giacoefficient(normalized, referenceSeries, gc);

% calculate general ranks
generalRank = sum(giaCoeff.*repmat(weights,numCase,1),2);
```

After initialization parameters, the first duty of the main function is to call *normalizeDataset* function that calculates and returns normalized dataset and *referenceSeries* variables. The MATLAB code of the *normalizeDataset* function is depicted in Figure 4. This function accepts *decisionMatrix* and *method* variables and calculates normalized variable by using the equations 2 and 3. Also it creates a new variable called *referenceSeries* by considering the method variable.

Figure4. Normalization Matrix

```

function [normalized, referenceSeries] = normalizeDataset(decisionMatrix,method)
[numCase, numVar] = size(decisionMatrix);
minSeries = min(decisionMatrix);
maxSeries = max(decisionMatrix);

normalized = zeros(numCase,numVar);
referenceSeries = zeros(1,numVar); % initialize reference series
for jdx = 1:numVar
if method(jdx) == 1 % Higher is better
for idx = 1:numCase
    normalized(idx,jdx) = (decisionMatrix(idx,jdx) - minSeries(jdx)) ...
        / (maxSeries(jdx) - minSeries(jdx));
end
    referenceSeries(jdx) = 1;
elseif method(jdx) == 0 % Smaller is better
for idx = 1:numCase
    normalized(idx,jdx) = (maxSeries(jdx) - decisionMatrix(idx,jdx)) ...
        / (maxSeries(jdx) - minSeries(jdx));
end
    referenceSeries(jdx) = 0;
end
end
end

```

The second function called in main function is *giacoefficient* and it calculates *gia* coefficients by using the equations 6 and 7. The inputs of this function must be normalized, *referenceSeries* and *gc*. However, the user should not worry about this function since it automatically called by main function. The MATLAB code of the *giacoefficient* is depicted in Figure 5.

Figure5. Calculating Grey Coefficients

```

function giaCoeff = giacoefficient(normalized,referenceSeries,gc)
[numCase, numVar] = size(normalized);
absData=zeros(numCase,numVar);
for i=1:numCase % Comparing reference series with normalized values
for j=1:numVar
    absData(i,j)=abs(normalized(i,j)-referenceSeries(1,j));
end
end
giaCoeff=zeros(numCase,numVar);
for i=1:numCase % Calculating gia coefficients
for j=1:numVar
    giaCoeff(i,j)=(min(absData)+gc*max(absData))/...
        (absData(i,j)+gc*max(absData));
end
end
end

```

3.4. Implementation of GRA Function

Athawale and Chakraborty (2011) reconsidered the problem described in Vahadania et al (2008). The problem is a supplier selection problem for a manufacturing organization. Five alternative suppliers and five criteria are considered in the problem. The values presented are in percentage values. The problem is originally presented in Vahadania et. al (2008) and it is solved by Balancing and Ranking Method. Athawale and Chakraborty (2011) solved the same problem by using Grey Relational Analysis. The dataset is presented in Table 1. The criteria weights are determined as $w_{ps} = 0.1942$, $w_{rc} = 0.2231$, $w_{tc} = 0.2779$, $w_{cq} = 0.1357$ and $w_{cr} = 0.1691$. All of the criteria has higher-is-better characteristic that implies that equation 2 must be used in normalization. The normalized values are depicted in Table 2. Grey relational coefficients are calculated by using

equation 7 and presented in Table 3. Finally, Grey Relational Grades are calculated using Equation 8 and presented in Table 4.

Table 1. Decision Matrix of Supplier Selection Problem

Supplier	Profitability of Supplier	Relationship Closeness	Technological Capability	Conformance Quality	Conflict Resolution
S_1	50,34	67,07	55,08	87,45	65,96
S_2	33,56	53,51	60,24	76,34	47,66
S_3	44,87	60,17	67,87	66,35	51,09
S_4	29,16	43,86	47,56	58,34	66,87
S_5	56,76	40,87	53,5	60,17	38,36

Table 2. Normalized Decision Matrix

Supplier	Profitability of Supplier	Relationship Closeness	Technological Capability	Conformance Quality	Conflict Resolution
S_1	0,7674	1	0,3703	1	0,9681
S_2	0,1594	0,4824	0,6243	0,6183	0,3262
S_3	0,5692	0,7366	1	0,2752	0,4465
S_4	0	0,1141	0	0	1
S_5	1	0	0,2925	0,0629	0

Table 3. Grey Relational Coefficients

Supplier	Profitability of Supplier	Relationship Closeness	Technological Capability	Conformance Quality	Conflict Resolution
S_1	0,6825	1	0,4426	1	0,94
S_2	0,373	0,4914	0,571	0,5671	0,426
S_3	0,5372	0,655	1	0,4082	0,4746
S_4	0,3333	0,3608	0,3333	0,3333	1
S_5	1	0,3333	0,4141	0,3479	0,3333

Table 4. Grey Relational Grades and Rankings

Supplier	S_1	S_2	S_3	S_4	S_5
GRG	0,7733	0,4897	0,6640	0,4522	0,4872
Rank	1	3	2	5	4

The same rankings can be obtained by calling following function in MATLAB directory where the above-mentioned dataset and functions available as in Figure 6. It should be noted that variables named *decisionMatrix*, *method* and *weights* must be available in the workspace. Please note that the results reported in Athawale and Chakraborty (2011) and in Figure 6 are same.

Figure 6. Calling *greyRelational* Function From Command Window in MATLAB Environment.

```
>> greyGrades = greyRelational(decisionMatrix,method,[],weights)

greyGrades =

    0.7733
    0.4897
    0.6640
    0.4522
    0.4872
```

4. RESULT AND CONCLUSION

Multi Criteria Decision Making tools are widely applied for business problems and it continue to use in literature. In this study, a function is proposed to calculate the grey relational grades. A modular approach is utilized in creating the function. The purpose of creating this algorithm is to ease the calculations of grey relational analysis. When a researcher needs to calculate grey grades, he/she should only run the GRA function that will dramatically increase the speed of the research. In

addition, researcher can focus on the problem without worrying about the mathematical calculation steps. The researchers in different fields can use the GRA function for different purposes in hybrid systems.

In this study, the functions are designed for teaching purposes. In other words, the functions are created without worrying about resource usage or the CPU time. Users are encouraged to design their own functions for their calculation purposes.

REFERENCES

- Ayrıçay, Y., Özçalıcı, M. ve Kaya, A (2013). Gri İlişkisel Analizin Finansal Kıyaslama Aracı Olarak Kullanılması: İmkb-30 Endeksindeki Finansal Olmayan Firmalar Üzerine Bir Uygulama. Kahramanmaraş Sütçü İmam Üniversitesi Sosyal Bilimler Enstitüsü Dergisi, 10(1):219-238.
- Athawale, V. M ve Chakraborty, S. (2008). Application of Grey Relational Analysis Method in Solving Supplier Selection Problems. The IUP Journal of Operations Management, 10(1):18-28
- Baş, M. and Çakmak, Z. (2012). Gri İlişkisel Analiz ve Lojistik Regresyon Analizi ile İşletmelerde Finansal Başarısızlığın Belirlenmesi ve Bir Uygulama. Anadolu Üniversitesi Sosyal Bilimler Dergisi. 12(3):63-82.
- Bektaş, H. ve Tuna, K. (2013). Borsa İstanbul Gelişen İşletmeler Piyasasında İşlem Gören Firmaların Gri İlişkisel Analiz ile Performans Ölçümü, Çankırı Karatekin Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi. 3(2):185-198.
- Çakmak, Z., Baş, M. and Yıldırım, E. (2012). Analysing Production Errors in an Establishment by Grey Relational Analysis and Correspondence Analysis. Süleyman Demirel Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi, 17(1):123-142.
- Li, G.D., Yamaguchi, D. Ve Nagai, M. (2007). Application of Grey-based Rough Decision-making Approach to Suppliers Selection. Journal of Modelling in Management. 2(2):131-142.
- Park, J.H., Park, I.Y., Kwun, Y.C. and Tan, X. (2011). Extension of the TOPSIS Method for Decision Making Problems under Interval-valued Intuitionistic Fuzzy Environment. Applied Mathematical Modelling. 35:2544-2556.
- Peker, İ. and Baki, B. (2011). Gri İlişkisel Analiz Yöntemiyle Türk Sigortacılık Sektöründe Performans Ölçümü. Uluslararası İktisadi ve İdari İncelemeler Dergisi. 4(7):1-18
- Şişman, B. and Eleren, A. (2013). En Uygun Otomobilin Gri İlişkisel Analiz ve Electre Yöntemleri ile Seçimi. Süleyman Demirel Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi. 18(3):411-429.
- Tayyar, N., Akcanlı, F., Genç, E. and Erem, I. (2014). BİST'e Kayıtlı Bilişim Ve Teknoloji Alanında Faaliyet Gösteren İşletmelerin Finansla Performanslarının Analitik Hiyerarşi Prosesi (Ahp) Ve Gri İlişkisel Analiz (Gia) Yöntemiyle Değerlendirilmesi. Muhasebe ve Finansman Dergisi. Ocak 2014:19-40.
- Vahdania, B., Zandieh, M. Ve Alem-Tabriz, A. (2008). Supplier Selection by Balancing and Ranking Method. Journal of applied sciences 8(19):3467-3472.