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AN INVESTIGATION ON HOW MUCH THE SECONDARY SCHOOL STUDENTS LEARN THE PLACE VALUE CONCEPT IN NATURAL NUMBERS[1]

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ABSTRACT

The purpose of this study is to find out how much the secondary school students can learn the concept of place value and what kind of mistakes those students who can not learn make and why they can not learn. The study universe constitutes a total of 176 students from 5th, 6th, 7th and 8th grade students who have been educated in various secondary schools of Van Province during the 2015-2016 academic year. Semi-structured interview forms have been used to collect data. We have interviewed each student individually and we have also interviewed their teachers individually about why the students can not learn. When the data we have obtained have been examined, it has been seen that the percentage of those who answered the questions related to the concept of place value correctly had been lower for each grade level. For this reason, when a hint is given a slight increase has been observed. As the success level of the students increases, the rate of error making naturally decreases. It has been understood from the interviews with the teachers that they had not used enough material for concreteness. It has been seen that this is a significant disadvantage in learning the place value concept correctly.

Key Words: Number, Mathematics Teaching, Place Value.

1. INTRODUCTION

Teaching of four operation in natural numbers is important in the primary school mathematics program between first year and fourth year. The natural numbers we use consist of a 10-based number system. In this system, 10 is considered to be the basic symbolic number and the expansion of every natural number is written as the power of 10. For this reason, this system is called the decimal number system (Demirtaş, 1986, Billstein et al., 1993, Hacısalihoğlu et al., 2000, Rappaport, 1966, Skemp, 1993, Sovchik, 1989; Artut and Tarım, 2006). The values of the numbers constituting the natural number at the place where they stand are called their place values. The concept of the place is very important and it constitutes a significant part of the arithmetic operations and the number system that the children deal with after learning it. The vast majority of errors that children make in arithmetic operations are due to the fact that these concepts are unknown or misunderstood. The decimal number system and the place value concept allow very small and very large numbers to be easily read and written with symbols (Busbridge and Womack, 1991, Deboys and Pitt, 1997, Reys et al., 1998).

Kamii and Joseph (1988), in the study of the teaching of summation in place valued and two-digit natural numbers, have asked the students for the digit value of a digit in the digit of a two digit-number. 33% of the students at the end of the 3rd grade and 50% of the students at the end of the 4th grade have answered this question incorrectly. It has been observed that these students had not considered the place value but had only considered the numerical value when evaluating the digit in its place.

Thompson (2000) has stated that the vast majority of the students could think of place value concept at very young ages, but has confused about this issue for a long time. Garlikov (2000) has investigated the studies on place value, and has stated that in America the students mostly could not learn the place value concept effectively.

[1] This study was carried out on October 18-21, Presented as an oral paper at the International Academic Research Congress.

In teaching the natural numbers and the four operation in these numbers, the place value concept is encountered as the most difficult subject for the students. (Artut and Tarım, 2006).

Since the place value concept is quite abstract, children's understanding is naturally difficult (Ross, 1986, Ross, 2002, Artut and Tarım, 2006, Nataraj and Thomas, 2007). The fact that the place value concept has not been learned well makes it difficult to learn many other concepts related to this concept. Ross, (2002) points out the importance of the place value concept to understand the natural numbers, comprehend different counting systems, thus to represent multiplicity in different bases, perform mathematical operations on mind and to forecast and understand multi-stepped operations.

Researches have revealed that the primary and the secondary school students have difficulties in understanding the place value and in performing and developing basic operations related to the place value. In fact, these studies have shown that most of the mistakes students make in arithmetic operations arise from the fact that this concept can not be fully learned (Arslan and Ubuz, 2009; Aydan, 2008; Bowers, 1996; Bowers, Cobb and McClain, 1999; Carpenter, Blume, Hiebert, Anick ve Pimm, 1982; Carpenter and Moser, 1984; Cauley, 1988; Cobb ve Wheatley, 1988; Cobb, Yackel and Wood, 1992; Artut and Tarım, 2006; Fuson, 1986; Fuson ve Briars, 1990; McClain, Cobb and Bowers, 1998; Steffe, Cobb ve von Glasersfeld, 1988; Arslan, Yıldız, Yavuz, 2011; Yenilmez and Demirhan, 2013; Baki and Güç, 2014; Uça, 2014; Tarım and Siyer, 2017; Şahin et al., 2014; Gürbüz et al., 2013; Gökkurt et al., 2013; Kleicmann et al., 2013; Stewart, 2013; Tanisli and Kose, 2013; Temel and Eroğlu, 2014; Özdeş and Kesici; 2014).

In studies conducted with teacher candidates as well as students, difficulties with the concept of place value have been encountered. For example, Chick (2003) asked teacher candidates to explain why one needs to add a zero at the end of the number when a number is multiplied by 10 in a study with 67 teacher candidates, 29 of whom are from secondary and 38 of whom are from primary education. He has stated that only 7 of them had given an acceptable answer and the rest of them either had answered incorrectly or could not answer. This shows that not only the students but also the teacher candidates have difficulties with the place value and can not learn this concept sufficiently.

The reason for the difficulty in understanding the value of the place is because of the fact that the writing language and the verbal language in our number system are not compatible with each other. We only need the digits from 0 to 9 when writing the numbers. However, in verbal language in addition to these digits, we use different words for ten and its folds (twenty, thirty, forty, ... hundred, thousand, million, ...). However, this usage leads to the disregard of the concept of place value. For example, the number 1235 is expressed in verbal terms as one thousand two hundred and thirty five. As it is seen, there is no phrase attributed to the place value in the verbal discourse (Arslan, Yıldız and Yavuz, 2011).

Another mismatch between the verbal expression of numbers and the way they are written is also because of the number 0 (Chambris, 2008). In daily language the number 0 is never mentioned but it is very important in written language. For example, we don't read 4002001 as four million zero zero two thousand zero zero one in verbal language, we instead read it as four million two thousand one. In this expression, the concept of place value is ignored (Arslan, Yıldız, Yavuz, 2011).

It can be said that the concept of number is obtained as a result of the studies carried out in order to determine and express the multiplicities or objects which exist in nature and which can somehow get into our lives. Counting, which can not be taken away from human life, has been made in different forms over the ages (binary, pentad, decimal, duodecimal, sexagesimal,... etc.). Today, the counting type commonly used in relationships among persons is the decimal counting. The counting that is first tried to be taught at schools is again decimal counting. The things that make the decimal counting system powerful are the place value and the use of ten symbols (0,1, ...,9) in order to be able to express a number. (Albayrak, İpek and Işık, 2006).

In view of the above explanations, it is thought that it is important to examine the competences of the teacher candidates who will primarily train the students and to determine the difficulties they are experiencing and the mistakes they are making in this regard, considering that they will contribute to understanding and eliminating the difficulties that the students have experienced concerning the place value concept (Artut and Tarım, 2013).

In the light of the above information, it is understood that the place value concept and numbers are based on one digit numbers 0, 1, 2, ..., 9, so all of our numbers are coming from these ten numbers. In this case, in teaching the concept of place value, it would be appropriate to discuss this topic with the following information in order to make them learn more easily.

By modeling our digits with blocks, after using one unit for 1 and two units for 2, …, nine units for 9, instead of using ten units for 10 by using the expression "ten units do not stand dispersed" we find a decimal by combining these ten units. And by showing our students a tenner we ask how many tenner there are and in response to the answer we have received, we write 1 to the board, then ask how many units there are, in response to the answer we receive, we write 0 next to the 1 on the board and we thus get number 10 as a result. We then by making a significant generalization teach the logic of the concept of place by saying that "ten units, ten tenners, ten centenarians, ... do not stand dispersed".

2. METHODOLOGY

2.1. Research Model

This research is a descriptive study in the screening model aiming to determine how much 5th, 6th, 7th, and 8th grade students of secondary school can learn the place value concept correctly, and what kind of mistakes those who can not learn make (Karasar, 1986; Artut and Tarım, 2006). For this purpose, semi-structured interview technique which is a qualitative research method has been used in the research.

2.2. Working Group

The sample of this research consists of 176 5th, 6th, 7th and 8th grade students from lower, middle and upper socioeconomic level educated in various middle schools in Van city of Turkey.

2.3. Data Collection Tool

Student interview form developed by Artut ve Tarım (2006) was used as data collection tool. This form is designed to determine the level of learning and the sources of the mistakes made related to the concept of place value. For this purpose, 15 counting bars that students can use and a card with number "12" on it are used.

2.4. Collection of Data

The interviews conducted with the students have been carried out in the following order and with the questions determined. The student have first been showed the number "12" and said, "Do you recognize this number? What number is this? " If the student knows the number, it is marked on the interview form and we have moved to the next question. At this stage, the question "Can you show this number using the counting bars?" was directed to the student. Information on whether the student could show this number by counting 12 bars was recorded in the form. Later, by showing "2" we have moved to the question "Can you show this part of the number using the counting bars?" The answer given by the student and the operation he has done are written on this form. In the next step, the student is directed to question "Can you show this part of the number using the counting bars?" by showing him "1". The result is marked on the form. At this stage, the numbers that the students have made with the bars are shown below.

Figure 1

Figure 2

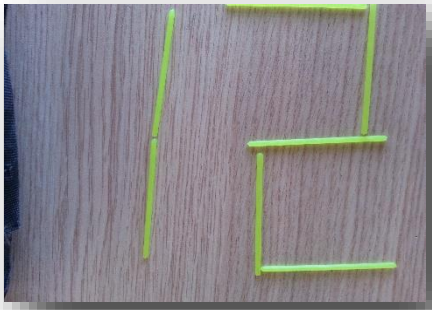

Figure 3

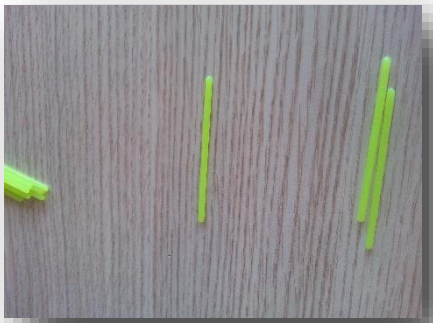

Figure 4

Figure 5

When the above figures are examined, figures 1, 2 and 3 arrive to the result that the student has to write figuratively with the number of bars they see and they are unaware of the conceptual knowledge of number "12", they only have mechanical knowledge, and they have learned numbers figuratively. In Figure 4, by evaluating concept information of each digit separately, he tried to show the number 12 by using bars as well. It has been concluded that these students have the concept knowledge of the digits but they are not aware of the decimal system and the concept of place. Figure 5 shows the readings of the numbers through bars, which is the conclusion that the situation of the group students concerning numbers is much behind in time. Note that in Figure 5 "on iki" means "twelve" in Turkish.

If the student has used 1 bar instead of 10 for 1 (Fig. 4), as a hint to reveal the way he perceives this point 1, by making explanation as "you have showed these bars (by showing all 12 counting bars at once) for this number and this many (by showing 2 bars) for this part as well; Alright, what do you think about the bars left over? Is there an oddity or is this a must? " The same question has been directed to the student again. The explanations from the student have been recorded exactly in the form.

After these explanations, it can be seen that some students could correctly show the number 12 as in Figure 6.

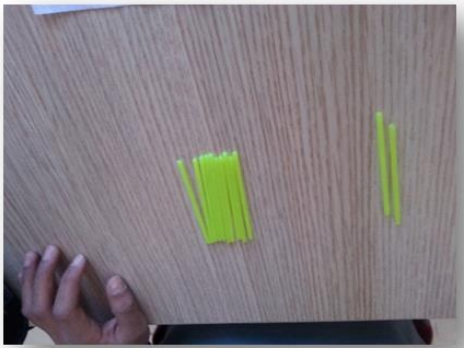

Figure 6

Individual interviews have been held with each student. The talks have lasted 30 minutes. The interview results have been recorded in the interview form.

3. FINDINGS

In the first question on the interview form, the answers given by candidates to the question "Do you know the number 12" are examined in Table 1 in terms of class level.

Table 1. Distributions of the answers given to the question "Do you know number 12" with respect to grade levels.

	Yes		No		Total	
Grade	Frequency(f)	%	Frequency(f)	%	Frequency(f)	%
5th grade	83	100	-	-	83	100
6th grade	42	100	-	-	42	100
7th grade	26	100	-	-	26	100
8th grade	25	100	-	-	25	100

When Table 1 was examined, all the students said that they knew the number 12. These results are anticipated as a result of waiting when students at this level are considered.

In the second question on the interview form, can you show the number "12" using the counting bars? The answers given by the candidates are examined in Table 2 in terms of class level.

Table 2. Distribution of the answers given to the question "Can you show the number 12 with counting bars?" with respect to grade levels.

	Yes		No		Tip by giving		Total	
Grade	f	%	f	%	f	%	f	%
5th grade	2	2.4	80	96.3	1	1.3	83	100
6th grade	3	7.1	37	88	2	4.9	42	100
7th grade	6	23	18	69.2	2	7.8	26	100
8th grade	6	24	16	64	3	12	25	100

When Table 2 is examined, the answers to the question "can you show the number "12" using the counting bars?" given by the candidates are listed with respect to their grade levels, it can be seen from Table 2 and Figures 1, 2, 3, 4 and 5 that a large majority could have not showed the number 12 with the counting bars or they have showed it incorrectly. When we look at these rates, we see that they are 96.3%, 88%, 69.2% and 64%, respectively. When we compare these ratios, it is seen that the failure to display the number 12 decreases as the class level increases.

After obtaining the data in Table 2, those students saying that they could show have been asked to show by giving them 12 counting bars. The answers given by the students are summarized in Figure 1, Figure 2, Figure 3, Figure 4, Figure 5 and Figure 6. Those who gave wrong answers as in Figure 1, Figure 2, Figure 3, Figure 4, and Figure 5, have answered the question "What do you think about the remaining bars?" as "the remaining bars are extra" or "I don't know" Thus these answers are handled in 2 categories. Table 3 gives the distribution of these answers with respect to grade levels.

Table 3. Distribution of the answers given by those students, showing number "12" with the counting bars incorrectly, about the remaining bars with respect to grade levels.

	I do not know		The remaining bars are extra		Total	
Grade	Frequency(f)	%	Frequency(f)	%	Frequency(f)	%
5th grade	49	61.25	31	38.75	80	100
6th grade	28	75.67	9	24.33	37	100
7th grade	11	61.11	7	38.89	18	100
8th grade	10	62.5	6	37.5	16	100

When we look at Table 3, it is seen that the students gave more "I do not know" answer for each grade level (61.25%, 75.67%, 61.11%, 62.5% respectively). It has been seen that at each class level, the different number of students have showed number 12 incorrectly, as in Figure 1, Figure 2, Figure 3, Figure 4 and Figure 5.

Table 4. Distribution of the answers given to the question "Can you show with the counting bars the value that number 2 in ones place is expressing? " with respect to grade levels.

	Yes		No		By Giving Hint		Total	
Grade	Frequency(f)	%	Frequency(f)	%	Frequency(f)	%	Frequency(f)	%
5th grade	76	92.8	6	7.2	1	1.2	83	100
6th grade	30	73.8	11	26.2	1	2.4	42	100
7th grade	26	100	-	-	-	-	26	100
8th grade	24	96	1	4	-	-	25	100

When the general data in Table 4 for each grade level of the students are examined we see that almost all students (92.8%; 73.8%; 100%; 96% respectively) have showed the value that the digit "2" in ones place is expressing in the number "12" when they have been asked to show by using the counting bars. While these have showed by counting two counting bars correctly, the rest have failed to do so.

Table 5. Distribution of the answers given to the question "Can you show with the counting bars the value that the number 1 in tens place is expressing?" with respect to grade levels.

	Yes		No		By Giving Hint		Total	
Grade	Frequency(f)	%	Frequency(f)	%	Frequency(f)	%	Frequency(f)	%
5th grade	19	22.9	48	57.8	16	19.3	83	100
6th grade	8	19	30	71.4	4	9.6	42	100
7th grade	4	15.4	22	84.6	-	-	26	100
8th grade	8	32	17	68	-	-	25	100

The answers given for the number in tens place with respect to grade levels are given in Table 5. When the answers given are examined, it is seen that the error rate is higher in intermediate classes, and is relatively less in the 5th and 8th grades (57.8% for 5th grade, 71.4% for 6th grade, 84.6% for 7th grade, and 68% for 8th grade).

The answers given by the students who responded incorrectly to the question "What do you think about the remaining bars?" have been handled in 2 categories. Some of the students replied as "the remaining bars are extra" or "I don't know". In Table 6 the distribution of these answers with respect to grade levels has been given.

Table 6. Distribution of the answers given by those, showing the place value of number "1" in tens place incorrectly with the counting bars, to the question about the remaining bars with respect to grade levels.

	I do not know		The remaining bars are extra		Total	
Grade	Frequency(f)	%	Frequency(f)	%	Frequency	%
5th grade	30	62.5	18	37.5	48	100
6th grade	20	66.7	10	33.3	30	100
7th grade	15	68.2	7	31.8	22	100
8th grade	12	70.5	5	29.5	17	100

When we look at Table 6, it is seen that students have responded with more "I do not know" answer for each grade level (62.5, 66.7, 68.2, 70.5, respectively). At each grade level, similarly, students were found to be inclined to express this with a single bar taking the numerical value of number 1 in tens place into account.

4. CONCLUSIONS AND RECOMMENDATIONS

This research has been conducted in order to determine the level at which secondary school students learn the place value concept and what kinds of mistakes those who can not learn make. When the results of this research have been considered, it has been determined that all students have recognized the number "12", but when the rate of students showing "12" by the counting bars is examined according to the class levels, while the rate of those who said "I can show" is 2.4% in the 5th grade 1.3% of them said that they could show if a hint is given, 96.3% of them said "no I can not show". Similarly, when the situation of the other classes is examined, the rate of those who said "I can show" is 7.1% in the 6th grade, 4.9% said "I can show you if you give me a hint", 88% said "no I can not show", the rate of those who said "I can show" is 23% of the 7th grade, 7.8% said "if you give a hint I can show", 69.2% said "no I can not show", the rate of those who said "I can show" is 24% in the 8th grade, 12% said that they could show if a hint is given, 64% said "no I can not show".

If we look after these statements at the number of students who can correctly show the number "12" with counting bars we see that Table 3 gives us enough information on this subject. When Table 3 is examined, only 3 students from 5th grade, only 5 students from 6th grade, only 8 students from 7th grade and finally only 9 students from 8th grade were able to show the number 12 correctly with the counting bars. In Table 3, when

the responses of the students who did not correctly show the number 12 with the counting bars were analyzed according to the class levels, 61.25% of the 5th grade students did not know, while 38.75% answered as "the remaining bars are extra". When we look at the other classes in the same way, these ratios appear as follows. 75.67% of the 6th grade students said "I do not know", while 24.33% of them said " the remaining bars are extra". 61.11% of the 7th grade students, said "I don't know", 38.89% of them said "the remaining bars are extra", and finally 62.5% of 8th grade students said "I do not know", 37.5% of them said " the remaining bars are extra". We conclude from these results that the students have a significant level of misconception about this concept.

When the answers given to the question asking to show the value that number 2 in ones place of "12" is expressing with the counting bars are examined with respect to the grade levels, 91.6% of the 5th grade students could show the correct value of the number 2 by showing two bars for the number 2, 1.2% of them can show correctly after giving a hint, and 7.2% of them said " no I can not show". When we look at other classes in the same way, 71.4% of 6th grade students can show the correct value of 2 by showing two bars for 2, 2.4% could show correctly after giving a hint, 26.2% said "I can not show", all of the 7th grade students showed the correct value of number 2 by showing two bars for the number 2, 96% of the 8th grade students showed the correct value of the number 2 by showing two bars for the number 2, and 4% of them said "no I can not show." We understand from these rates that our students know the place value of the digits between 0-9 very well.

When we examine the answers given to the question asking to show the value that number 1 in tens place of "12" is expressing with the counting bars with respect to the grade levels, we see that 22.9% of the 5th grade students showed the value that number 1 is expressing correctly by showing ten counting bars, 19.3% of them were able to show correctly after giving a hint, 57.8% of them said "no I can not show" or they showed incorrectly. When we looked at the other classes in the same way, 19% of 6th grade students could show the correct value of 1 by showing ten bars for number 1, 9.6% could show correctly after giving a hint, 71.4% of them either said no they could not show or they showed incorrectly, 15.4% of the 7th grade students showed the correct value of 1 by showing ten bars for number 1, 84.6% said either they could not show or they showed incorrectly, 32% of 8th grade students showed the place value of 1 correctly by showing ten counting bars, and 68% either said they could not show or they showed incorrectly. We understand from these rates that while the students are very good at determining the place value of the digits between 0-9 they were unable to show the same success on the concept of tenner, they even became too unsuccessful.

In Table 5, when we examine the answers given by those, showing the value that the number 1 in tens place of the number "12" is expressing with the counting bars incorrectly, about the remaining bars with respect to grade levels, 62.5% of the 5th grade students said they did not know, and 37.5% of them said that the remaining bars were extra. Similarly, when we look at the other classes these ratios appear as follows. 66.7% of the 6th grade students said they did not know, 33.3% of them said the remaining bars were extra, 68.7% of the 7th grade students said they did not know, 31.8% of them said the remaining bars were extra, and finally 70.5% of the 8th grade students said they did not know, and 29.5% of them said the remaining bars were extra. These results tell us that the students have significant level of misconception about the concept of tenner.

This study has shown us that even middle school students, whom we expected to know the concept of place value very well, have very serious deficiencies about this concept. One of the reasons why the level of correct answers to the questions about the concept of place value is so low may be related to the level of knowledge teachers have about this concept. Broadbent (2004) emphasized that teachers should develop their knowledge of the decimal system before teaching this concept to their students. To do this, the teachers have to focus on the features of the number system. (Artut and Tarım, 2016).

As a result of the researches carried out, it is thought that it is important to examine the competences of teacher candidates who will primarily train them and to determine the difficulties they are experiencing and the mistakes they make, considering that students will contribute to understanding and eliminating the difficulties they have experienced in the concept of place value. (Artut and Tarım, 2013).

When the results of this study are evaluated it can be suggested that the studies related to the place value concept in the education of the teacher candidates and noticing the possible student mistakes in the teaching of the subjects related to different counting systems, explaining the reasons for those mistakes and error analysis must be considered. It may be advisable to conduct empirical studies using different teaching methods, e.g. cooperative learning method, or using different teaching materials (tenner counting blocks, place value tables, scorecards, legos). However, it may be advisable to work with larger sample groups and establish robust evidence for generalizations. (Artut and Tarım, 2013).

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